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Comment on "Accelerated Detectors and Temperature in (Anti) de Sitter Spaces"

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Abstract

It is shown how the results of Deser and Levin on the response of accelerated detectors in anti-de Sitter space can be understood from the same general perspective as other thermality results in spacetimes with bifurcate Killing horizons.

A detector with linear acceleration a in the Minkowski vacuum sees a thermal bath at the temperature $T_U = a/2\pi$ [1], while an inertial detector in de Sitter (dS) space of radius R sees a thermal bath in the de Sitter vacuum at the temperature $T_{GH} = 1/2\pi R$ [2]. What does an accelerated detector see in de Sitter space? This detector also sees a thermal bath, but at the temperature[3, 4, 5]

$$T_{dS} = (R^{-2} + a^2)^{1/2}/2\pi. \tag{1}$$

Deser and Levin (DL) recently showed [5] that the same formula with $R^2 \to -R^2$ gives in anti-de Sitter (adS) space the temperature seen by some uniformly accelerated detectors in any of three vacuum states, while the temperature for some other uniformly accelerated detectors vanishes! (In the adS case the class of accelerated world lines yielding (1) has acceleration bounded below by R^{-1} so the argument of the square root is bounded below by zero.)

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In the dS case DL obtain the result (1) by viewing de Sitter space as a timelike hyperboloid embedded in five dimensional Minkowski space, and invoking the fact [6] that the Wightman function for a conformally coupled massless scalar field in dS is just the induced Wightman function from the 5d Minkowski vacuum. Since the uniformly accelerated worldlines in de Sitter are also uniformly accelerated in the 5d Minkowski embedding space, the detector response is the same as in the 5d Minkowski vacuum, which appears as a thermal state at the Unruh temperature $T = a_5/2\pi$. The result (1) then follows from

$$a_5 = (R^{-2} + a^2)^{1/2}. \tag{2}$$

AdS space is the hyperboloid $(z^0)^2 - (z^1)^2 - (z^2)^2 - (z^3)^2 + (z^4)^2 = R^2$ in a flat 5d "ultra hyperbolic" embedding space. DL compute the detailed form of the transition rate in first order perturbation theory for a detector on two classes of uniformly accelerated worldlines in adS, (i) circles $(z^0)^2 + (z^4)^2 =$ const at fixed (z^1, z^2, z^3), and (ii) hyperbolas $(z^0)^2 - (z^1)^2 = \text{const} < 0$ at fixed (z^2, z^3, z^4). Since adS is not globally hyperbolic there is no unique "vacuum" state, but DL examine three different choices for the adS quantum field state corresponding to different boundary conditions at infinity [7]. For all of these states the detector transition rate vanishes for worldlines of class (i), and for those of class (ii) the rate has a Planck factor at the temperature (1), although the actual rate in two of the three cases has an energy dependent prefactor.

The purpose of this comment is to indicate another way of arriving at some of the same conclusions which emphasizes the role of the bifurcate Killing horizon, the global thermal nature of the states, and the connection with related results in Minkowski, de Sitter, and Schwarzschild spacetimes. Let us begin with de Sitter space, which has no globally timelike Killing field. A boost-like dS Killing field vanishes on the equatorial S^2 of an S^3 spatial slice. This S^2 is the bifurcation surface of the Killing horizon. In terms of the Hamiltonian H_B that generates evolution along this Killing field, the density matrix of the de Sitter vacuum restricted to one side of the Killing horizon takes the form[8, 9]

$$\rho = Z^{-1} \exp(-2\pi H_B). \tag{3}$$

In writing (3) the Killing field has been taken to be $\xi = \partial/\partial\theta$, where on the Euclidean section the range of the dimensionless variable θ is 2π. Thus the de Sitter vacuum is thermal at the dimensionless temperature $T = 1/2\pi$.

To determine the temperature T' seen by an observer on any orbit of this Killing field note that if ξ has norm N on this orbit, then the rescaled Killing field $\xi' = N^{-1}\xi$ has unit norm there, so the Hamiltonian appropriate to this observer scales as $H'_B = N^{-1}H_B$, so that the effective temperature scales as

$$T' = N^{-1}T = N^{-1}/2\pi. \tag{4}$$

This is just the Tolman redshift. It is easy to see that on a Killing orbit of acceleration a the norm N of ξ is just the inverse of the "five"-dimensional acceleration (2), so (4) yields (1).

In adS there *is* a globally timelike Killing field which generates rotations in the z^0z^4 plane of the 5d embedding space. This is τ-translation in the intrinsic 4d line element (9) of [5]. A detector following an orbit of this Killing field will remain unexcited in the ground state of the Hamiltonian H_τ that generates this symmetry. The three states considered by DL are ground states for H_τ corresponding to different boundary conditions at infinity, hence there is no detector excitation for these orbits.

There is also in adS a boost-like Killing field with bifurcate Killing horizon. I will now argue that the ground state of H_τ is a thermal state relative to the Hamiltonian H_B generating the boost-like Killing symmetry. On the covering space, with the timelike direction unwrapped, the bifurcation surface is an infinite collection of hyperbolic planes. In spite of the lack of global hyperbolicity, the usual Euclidean path integral construction[8] of the wave-functional for the ground state of H_τ on a spatial slice through one of the bifurcation surfaces can still be used. I don't know of anywhere this path integral is carried out explicitly for the adS case, but it seems clear that it depends upon the boundary conditions imposed at infinity and can yield any of the states considered in [7]. As in Minkowski, de Sitter, and Schwarzschild spacetimes, slicing this path integral in "polar" coordinates adapted to the Euclideanized boost-like symmetry reveals that, restricted to one side of the bifurcation surface, the state is a thermal density matrix of the form (3).

What temperature do different boost-like Killing observers see in adS? The norm of the Killing field on a Killing orbit of acceleration a can be obtained from the dS case by analytic continuation of R^2 to negative values, so (4) yields the result of DL,

$$T_{adS} = (-R^{-2} + a^2)^{1/2}/2\pi, \tag{5}$$

which is just (1) with $R^2 \to -R^2$. The acceleration of the adS Killing orbits approaches the constant R^{-1} at infinity (unlike in Rindler or Schwarzschild spacetime where it vanishes). Thus T_{adS} (5) vanishes at infinity, even though the Killing orbits are still accelerating there. This is because, as (4) shows, it is not the acceleration but the norm of the Killing field that determines the locally observed temperature in the state defined by the Euclidean path integral. The norm of the relevant adS Killing field diverges at infinity, so the observed temperature vanishes there, as in the Minkowski vacuum in Rindler space. In Schwarzschild spacetime, conversely, the norm of the Killing field is finite at infinity, so the local temperature is nonzero there even though the acceleration vanishes. This is just the Hawking temperature of the Hartle-Hawking state at infinity.

I have argued that the ground state of H_τ is a thermal state relative to the Hamiltonian H_B generating the boost-like Killing symmetry. This is a general conclusion which applies (at least formally) to *any* field theory, including interacting ones. Nevertheless, detectors will respond differently in the ground states corresponding to different field theories. Even in free field theory, boundary conditions will influence the detector response due to the different nature of the modes that generate the Hilbert space. The calculation of Deser and Levin reveals precisely how the different boundary conditions affect the detector response for a conformally coupled massless free field, which is something that cannot be obtained from (3) without an explicit construction of the operator H_B in each case.

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